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The Geometric Primitive MatOrus

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Abstract- This paper explores the potential of forming hierarchical structures using a geometric primitive, that we called MatOrus, which is a 3D closed surface formed from a Bézier circle. Several MaTorii (a set of MatOrus) can be systematically assembled into variations of interlocking structures, therefore multiple MaTorii can be used as basic elements to build larger and complex geometric structures. We investigate a method that can generate the geometry proposed using a parametric coding so that the several shapes can be created easily. We also propose several forms of integrating the basic form in order to obtain aesthetic geometry that can be used in fields such as sculpture and architectural design. Our results show that the method requires minimum of memory source, a few number of operations and it is effective in procedural parameterizing.
Keywords – Computational Geometry, Bézier curves, parametric coding, tridimensional design

I. INTRODUCTION

One of the most significant dimensions of parametric modeling software is representing variation in design. This works best when variation is continuous and distinctly different alternatives are not part of the model. Patterns are used as constructs to collect, organize, and communicate regularities in designer use of parametric design tools. In this article a geometric primitive is introduced, MatOrus that is created from a Bézier curve handled with four edges. The primitive is deformed in two parallel edges (the control handles) and filled to form a surface. This is typically smooth, but also has a shape which can be controlled by the designer. It is therefore flexible such as the natural quadrics (spheres, planes, circular cylinders and cones). Other work suggest geometric primitives with a polygonal approach [7] which differ from our focus, the usage of curves and surfaces.

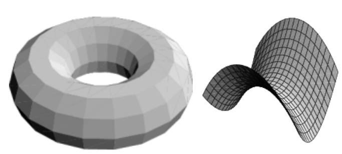
The designer can specify a shape in several ways, keeping the control points of the Bézier surfaces, or convert the primitive into a mesh. The designed surface approximates the given control mesh similar to a Torus, whilst guaranteeing a certain level of smoothness. The basic idea is to provide a mechanism whose users can exploit their abilities to create structures for sculpture and architecture.

Main contributions include the following: a strategy for constructing a MatOrus, a merge of a Torus with Bézier curves. A mathematical representation of such geometric primitive when combining with other similar primitives. We propose a set of MatOrus placed together on different operations (translations and rotations) to achieve aesthetic designs. Some scripts are implemented to illustrate the variety of shapes that can be created.

The paper is organized in the following manner: The next Section presents the concept of a Bézier curve and Section III describes the theoretical basis to create MatOrii. Section IV details the 3D representation when two or three MatOrii are joined by applying geometric transformations. In the Scripting Section, a scripting code is explained; rotation and translation operations are performed. Finally, we conclude with a review and analysis of the findings of the research.

II. BASIS OF THE MATORUS

The need to compute curved 3D surfaces with the assistance of computer numerically controlled manufacturing was the very impetus for the first computer aided design software over half a century ago, principally developed in the automotive and aerospace industries. Parametric design is an emerging research issue in the design domain. In this work, our main focus lies on curves and surfaces, but our point of view is algorithmic. We extent a Bézier circle to an arbitrary topology and that generalization can produce surfaces with similar quality to the subdivision representation. We believe that generalization is also interesting from a theoretical point of view. The fast rise of computing in the last decades has made it possible to work massively and in a graphic and intuitive way with mathematical representations of tridimensional geometry, such as the NURBS, Bézier, and Splines. These organic surfaces of free shapes defined by vectorial curves have allowed access to a rapid generation of complex shapes with a minimum amount of data and of specific knowledge.



(a) (b)

Figure 1. (a) Torus and (b) Elliptic Hyperboloid

The geometric primitive to start with is the circle $(x\text{-}xc)^2 + (y\text{-}yc)^2 = r^2$, where (xc, yc) is the center and r is the radius. The circle can be transformed to other figures in 3D such a as the sphere of radius r centered at the origin is given in Cartesian coordinates by

$$x^2 + y^2 + z^2 = r^2, \qquad (1)$$

which is a special case of an Ellipsoid or a Torus (Figure 1a), that is a surface of revolution generated by revolving a circle in three dimensional space about an Z axis coplanar with the circle (R-square root$(x^2 + y^2)$ $)^2 + z^2 = r^2$, where R is the radius of the inner circle. The parametric equations of the Torus [4] are given as follows

$x = (c + a \cos v) \cos u$, $y = (c + a \cos v) \sin u$, $z = a \sin v$

And the Elliptic Hyperboloid (Fig 1b) was constructed in X and Y as following

$$z= x^2/b - x^2/a \qquad (2)$$

However, our idea was to use the curve established by the Hyperbolic Paraboloid merged with the Torus. Since the suitability to manipulate the circle we thought in control points, specifically the handlers of the Bézier curves. Rather than forming specific geometry such as the Sphere or Torus, the circle allows to apply affine transformations and deformations in order to obtain new geometry in 2D and 3D.

Bézier curves are often used to generate smooth curves because they are computationally inexpensive and produce high-quality results. How can the circle be approximated with Bézier curves? The standard approach is to divide the circle into four equal sections, and fit each section to a cubic Bézier curve. This reduces the problem to a matter of fitting a cubic Bézier curve to a right circular arc. The standard approach imposes the following constraints:

- The endpoints of the cubic Bézier curve must coincide with the endpoints of the circular arc, and their first derivatives must agree there.
- The midpoint of the cubic Bézier curve must lie on the circle.

The general formula of cubic Bézier curve is as follows:

$$B(t) = (1-t)^3 P_0 + 3(1-t)^2 tP_1 + 3(1-t)^2 t^2P_2 + t^3P_3 \qquad (3)$$

With $t \in [0,1]$. The second constraint implies that:

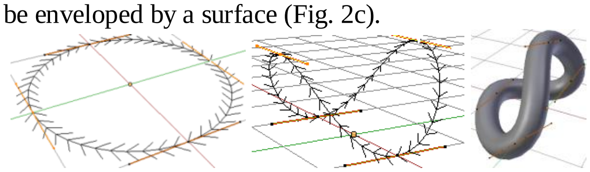
$P_0 = (0,1)$, $P_1 = (c,1)$, $P_2 = (1,c)$, $P_3=(1,0)$

And provides the values of c [1]:

$c= (4/3)(\text{square root}(2)-1) \approx 0.5522847498$

III. FROM THE BÉZIER CIRCLE TO THE MATORUS

A Bézier circle can be manipulated by taking the control points of its four edges and applying affine transformations. We start creating a Bézier circle with four control points (Fig. 2a). These control points can be translated as shown in Figure 2b. This new shape can be enveloped by a surface (Fig. 2c).



(a) (b) (c)

Figure 2. (a) The Bézier circle in original form (b) the shape obtained after moving two control points (c) the shape filled by a surface

Tubular surfaces are built from a base curve that defines its length and another curve which is displaced over the first curve as illustrated in Figure 3.

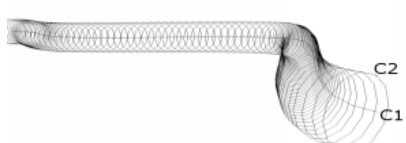


Figure 3. A tubular surface is formed by two curves, the base curve C1 defines the length nad direction, and C2 that is displaced along the first curve to define the surface

Bauer and Plothier [2] presented a method to build the curve from a surface, using the minimal squares method in order to find the best approximation to a cylinder. In [3] the geometric construction method of curved folded tubular and cellular structures using a space curve was shown. The Bézier circle can be converted into a kind of Torus that we called MatOrus by moving up two parallel control points.

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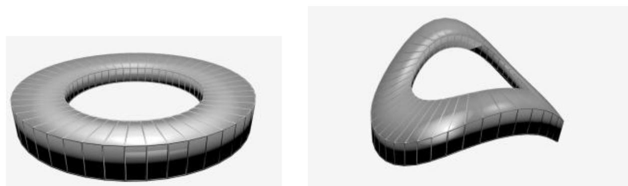
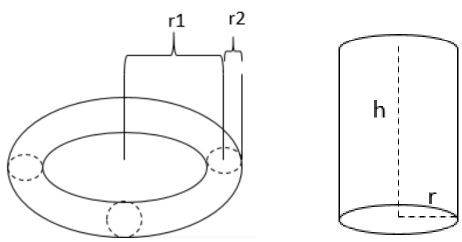
The last step consists on converting the rounding surface from circle to a rectangle to give our specific feature; this can be done by employing a bevel object operation (Fig. 4a). In this sense, the MatOrus is similar to a disk, or can be seen as a small cylinder (Fig. 5). However, the MatOrus is really a disk whose measures can be computed with the difference of two cylinders. Let r_1 be the radius of the big cylinder and r_2 be the radius of the small cylinder, the volume of the disk equals $V= \pi h(r_1^2-r_2^2)$, and analogously with the surface area. The other consideration is that our primitive is not exactly a disk, it is a curved disk (Fig. 4b). The volume and surface area can be calculated by summing the squares (or cubes) that form the disk. Note, the MatOrus is a deformed disk, so that we would think about adjusting the measures. However, the deformation was created before the construction of the surface area: the Bézier circle is created, deformed, and finally the curve is filled by the squares, forming a surface. Therefore, the volume can be computed by summing the volumes of the squares utilized or using a revolution method. Our object is a rigid body due to the volume is preserved, it cannot allow deformations, and however it can be adjusted applying affine transformations.



(a) (b)

Figure 4. (a) The Torus is formed by joined squares rounding a circle. (b) The disk is deformed

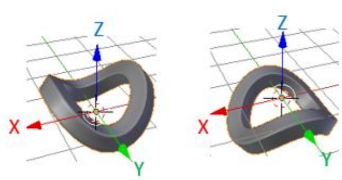


(a) (b)

Figure 5. (a) The volume and surface area of a Torus with inner radius r1 and ring radius r2: $S=4\pi^2 r1r2$, $V=2\pi^2 r1r2^2$. (b) The volume and the surface area of a cylinder $S=2\pi(r+h)$, $V=\pi r^2 h$.

IV. CONSTRUCTING AESTHETIC SHAPES

Parametric design is understood as a process where a description of a problem is created using variables. By changing these variables, a range of alternative solutions can be created. Our aim is to use the MatOrus to construct structures that serve as inspiration for buildings or sculptures. By applying different methods to deform a Torus we could obtain interesting geometric primitives. We first take a MatOrus as a basis, for instance the MatOrus depicted in Figure 6 is oriented towards the Z-axis.



(a) (b)

Figure 6. (a) The MatOrus oriented towards Z-axis (b) the MatOrus rotates 180 degrees over the X-axis

Let ζ be a MatOrus, then we can obtain more shapes by joining two or more MatOrii, ζ_1 and ζ_2, by using affine transformations, basically rotation and translation. Let $\zeta_i^{\,x}$ be a MatOrus oriented towards X-axis and $-\zeta_j^{\,x}$ be its opposite MatOrus which remains the orientation, but it is rotated 180 degrees. Then we can join these two MatOrii as depicted in Figure 7a. The operator $R_{y,45}$ applies a rotation of 45 degrees over Y-axis (Fig. 7b).

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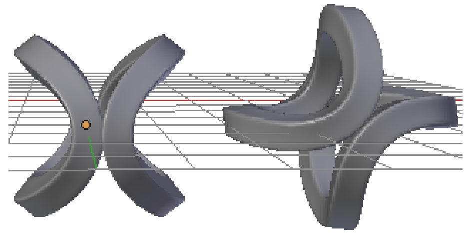
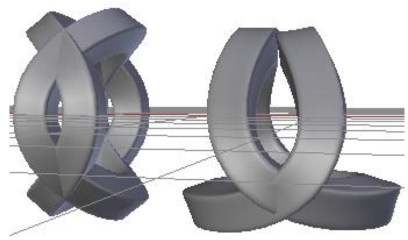


(a) (b)

Figure 7. (a) Vertical joins of two MatOrii $-\zeta_0^{\ x}\ \zeta_1^{\ x}$. (b) The same shape rotated 45 degrees in Y-axis, $R_{y,45}(-\zeta_0^{\ x}\ \zeta_1^{\ x})$.

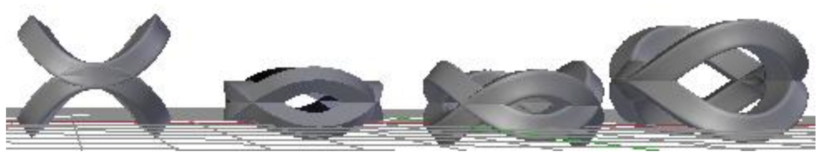
Operator $T_{x,-r}$ translates –r units along the X-axis. A new shape is formed by intersecting the bottom part of both MatOrii (Fig. 8).



(a) (b)

Figure 8. (a) Two vertically MatOrii joined with intersection $-\zeta_0^{\ x}\ T_{x,-2r}\ (\zeta_1^{\ x})$ (b) Two vertically MatOrii joined in Y-axis in order to have intersection in the bottom part only: $R_{y,25}\ (-\zeta_0^{\ x})\ R_{y,-25}(T_{x,-2r}\ (\zeta_1^{\ x}))$.

We can place two MatOrii together in plane XY (the horizontal plane) oriented towards Z-axis (vertical axis). Fig. 9.



(a) (b) (c) (d)

Figure 9. (a) Two horizontally MatOrii oriented towards Z-axis (a) $T_{z,-r}\ (-\zeta_0^{\ z})\ \zeta_1^{\ z}$, (b) $T_{z,-2r}\ (-\zeta_0^{\ z})\ \zeta_1^{\ z}$, (c) $T_{z,-3}\ (-\zeta_0^{\ z})\ \zeta_1^{\ z}$, (d) $T_{z,-4}\ (-\zeta_0^{\ z})\ \zeta_1^{\ z}$

V. SCRIPTING

From now on, other structures can be created by the procedural method using a script. In this section we suggest a script to form complex structures with the MatOrus. We employ the 3D software Blender [6] to make scripts in Python. Three main functions are constructed. The first function makes a copy of the MatOrus basis, create. The rot function performs an Euler rotation operation, specifying the axis and the degrees. In the case we require a rotation in two axis, the rot2axis function is used. The translation function can be performed by using one or two axis: trans2axis. Now we can construct some interesting structures by calling the created functions. In the first structure (Fig 10) four MatOrii are used. The first MatOrus is called MatOrus.001 and does not translate, but rotate and the other MatOrii need both, translation and rotation. The second structure also requires 4 MatOrii and only rotation operation in one axis is performed, Figure 11. The third structure also requires 4 MatOrii and only rotation operation in one axis is performed (Fig. 12). The fourth structure is formed with three MatOrus (Fig. 13).

```
i=1
create(i)
rot(i, "MatOrus.001", -pi/4, "Y")
i=2
create(i)
rot(i, "MatOrus.002", pi, "Y")
trans(i, "MatOrus.002", 1.0, "Z")
i=3
create(i)
rot2axis(i, "MatOrus.004", pi/2, "X", pi/2, "Z")
trans2axis(i, "MatOrus.004", -1.0, "X", -1.0, "Z")
i=4
create(i)
rot2axis(i, "MatOrus.004", pi/2, "X", -pi/2, "Z")
trans2axis(i, "MatOrus.004", 1.0, "X", -1.0, "Z")
```

Figure 10. The first structure

```
i=1
create(i)
i=2
create(i)
rot(i, "MatOrus.002", pi/2, "X")
i=3
create(i)
rot(i, "MatOrus.003", -pi/2, "X")
i=4
create(i)
rot(i, "MatOrus.004", pi, "X")
```

Figure 11. The second structure

```
i=1
create(i)
i=2
create(i)
rot(i, "MatOrus.002", pi/2, "X")
i=3
create(i)
rot(i, "MatOrus.003", -pi/2, "X")
i=4
create(i)
rot(i, "MatOrus.004", pi, "X")
```

Figure 12. The third structure

```
i=1
create(i)
i=2
create(i)
rot(i, "MatOrus.002", 2*pi/3, "Z")
i=3
create(i)
rot(i, "MatOrus.003", 4*pi/3, "Z")
```

Figure 13. The fourth structure

VI. CONCLUSION

We have introduced a geometric primitive that we called MatOrus, a simple element with great potential for building composite structures. This is formed from a Bézier circle that is transformed and filled with a 3D mesh. We propose a set of geometric structures and their algebraic representation. The MatOrus is oriented towards an axis, to define its placement (horizontal, vertical or any other plane), and the curvature is defined by the Bézier curve control points. The Surface and Area of the volume determined by the MatOrus are essential measures to take into account.

The MatOrus is feasible to create hierarchical structures using a set of them. Generative rules for the engagements of MatOrii enable efficient means for the analysis and synthesis of forms that could be built with MatOrii. Its usage in fields such as architecture or sculpture is yet to be discovered. Various materials, details, sizes of MatOrii should be tested for practical applications.

We believe that the combination of interactive and procedural modeling is a significant boost to artist's productivity and a great complement to existing modeling tools. In an era in which designers commonly seek to both add geometric sophistication to build forms as well as to reduce the embodied energy of their projects, lessons from the work outlined in this paper have significant implications. As a further work we suggest to create more structures with the MatOrus and to make structural design for engineering.

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